FINAL REPORT

on

DYNAMIC RESPONSE OF THE ELASTIC-PLASTIC FINITE ELEMENT*

by

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1.0 A brief discussion follows as to the theoretical models pursued in this project and the structural idealizations thereof.

1.1 Constitutive Equations:

There are indeed a large number of possibilities of considering the stress strain relations in the elastic-plastic region. For instance the Ramberg-Osgood relationship is valid for a unidirectional stress situation and is given by:

$$e = \frac{\sigma}{E} + k\sigma^n$$

On the other side of the spectrum are the flow laws relating stresses and strain rates expressed in three dimensions. e.g.

$$\dot{e}^p_{ij} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{ij}}$$

Deformation laws relating stresses and strains are a mathematical approximation to the above and their validity can only be discussed in context of the accuracy of a particular problem. One such relationship is the Prandl-Reuss equations relating the plastic strain increment to the instantaneous stress deviation.

$$de^p_{ij} = d\lambda \; S_{ij}$$

It can be shown that this set of equations implies the von Mises yield criterion (1). However, there is a benefit, as Marcal has noted (2), in incorporating explicity the von Mises yield criterion or rather its implicit differential in the Prandtl-Reuss equations. The resulting equations are given below:

$$de_x = \frac{3}{2} \frac{S_x}{\bar{\sigma}} d\bar{e}_p + \frac{1}{E} d\sigma_x - \frac{\gamma}{E} d\sigma_y - \frac{\gamma}{E} d\sigma_z$$

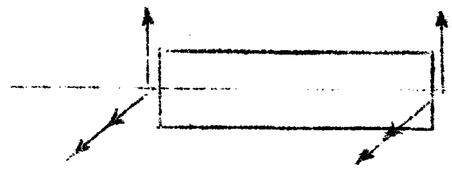
$$de_y = \frac{3}{2} \frac{S_y}{\bar{\sigma}} d\bar{e}_p - \frac{\gamma}{E} d\sigma_x + \frac{1}{E} d\sigma_y - \frac{\gamma}{E} d\sigma_z$$

$$de_z = \frac{3}{2} \frac{S_z}{\bar{\sigma}} d\bar{e}_p - \frac{\gamma}{E} d\sigma_x - \frac{\gamma}{E} d\sigma_y + \frac{1}{E} d\sigma_z$$

$$de_{xy} = 3 \frac{\sigma_{xy}}{\bar{\sigma}} d\bar{e}_p + \frac{1}{G} d\sigma_{xy}; \quad de_{yz} = 3\frac{\sigma_{yz}}{\bar{\sigma}} d\bar{e}_p + \frac{1}{G} d\sigma_{yz}; de_{zx} = 3\frac{\sigma_{zx}}{\bar{\sigma}} d\bar{e}_p + \frac{1}{G} d\sigma$$

These constitutive equations were considered most adequate for the present purposes.

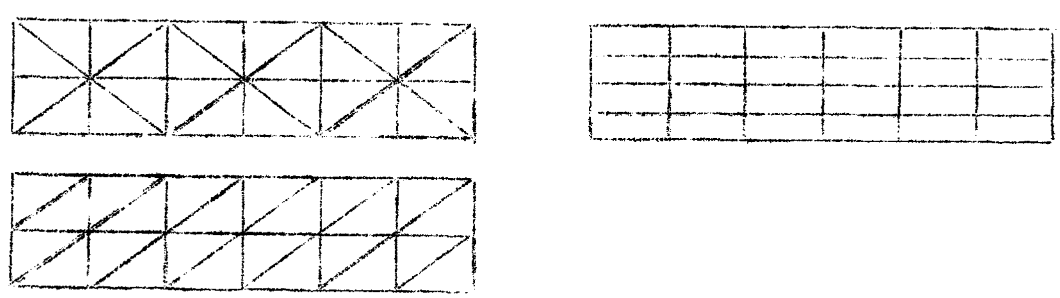
1.2 Finite Element:



First a beam element with and without axial loads was considered. Here a uniaxial stress strain relationship would be considered, e.g. elastic ideally plastic or Ramberg-Osgood type of relationship. It may be noted however, that for a stiffness formulation, this approach lends itself to complexities. A modification of this approach has been tried by using layered flange elements which allow for the fact that a given section the strains along a horizontal line will be the same (3). It is felt that this attempt is not too successful and that the results are too restrictive.

It was therefore thought that a certain degree of generality could be retained if a beam or a beam element may be thought of as an assembly of a series of finite elements. The individual elements represent a continuum in a state of plane stress. The resultant forces and displacements at the ends of the elements would then be interpreted to formulate the stiffness matrix of the beam or the beam element.

Triangular and rectangular elements were considered in this project; the arrangements were as shown below:



The displacement functions used were:

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y \qquad v = \alpha_4 + \alpha_5 x + \alpha_6 y$$

for the triangular elements and

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \qquad v = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy$$

for the rectangular elements

A computer program was written and debugged to find the resulting displacements, etc. due to static loads applied to the above arrangements of the finite elements. The modified Prandtl-Reuss equations are incorporated in the program.

1.3 Dynamic Response and Free Vibrations:

With the development of the static program it would not be difficult to obtain a time dependent response of a given system to a given input of time variable loading if sufficiently small increments of time are considered during which the "static" analysis is done.

However, the study of free vibrations for an elastic-plastic element, that is, formulation in terms of a mass matrix and of how this is affected by elastic plastic stress strain relationships remains inconclusive.

2.0 Student participation:

Two senior students, now graduates, have worked almost for the entire time of this project and in addition another graduating senior assisted on the programming aspects of the project during the summer. It is believed that the project has stimulated their professional interests and has exposed them to the specific tools namely the finite element method and computer programming.

References:

(1) Mendelson, A., Plasticity: Theory and Application, McMillan, New York, 1968, p.104

(2) Marcal, P. V., A Stiffness Method for Elastic-Plastic Problems, International Journal of Mechanical Sciences, Pergamon Press, 1965, Vol 7, pp 229-238.

(3) Salus, W. L., Ip, Ching-u, and VanDerlinden, J. W., Design Considerations of Elastic Plastic Structures Subjected to Dynamic Loads, Proceedings of the AIAA/ASME 11th Structures, Structural Dynamics and Material Couterence, 1970, pp 145-153